

2014 Mathematical Methods (CAS) Written examination 1 solutions

Question 7

$$f(x) = \int (2 \cos(x) - \sin(2x)) dx = 2 \sin(x) + \frac{1}{2} \cos(2x) + c$$

$$\frac{1}{2} = 2 \sin\left(\frac{\pi}{2}\right) + \frac{1}{2} \cos(\pi) + c$$

$$\frac{1}{2} = 2 - \frac{1}{2} + c$$

$$-1 = c$$

$$\therefore f(x) = 2 \sin(x) + \frac{1}{2} \cos(2x) - 1$$

Question 8

a. $\int_0^m \frac{1}{5} e^{-\frac{x}{5}} dx = \frac{1}{2}$

$$\left[\frac{1}{5} \times -5 e^{-\frac{x}{5}} \right]_0^m = \frac{1}{2}$$

$$-e^{-\frac{m}{5}} + 1 = \frac{1}{2}$$

$$\frac{1}{2} = e^{-\frac{m}{5}}$$

$$\log_e\left(\frac{1}{2}\right) = -\frac{m}{5}$$

$$m = 5 \log_e(2)$$

b. $\frac{\Pr(X < 1 \cap X \leq m)}{\Pr(X \leq m)}$

$$= \frac{\Pr(X < 1)}{\Pr(X \leq m)}$$

$$= \frac{\left[-e^{-\frac{x}{5}} \right]_0^1}{\frac{1}{2}}$$

$$= \frac{-e^{-\frac{1}{5}} + 1}{\frac{1}{2}}$$

$$= \frac{-e^{-\frac{1}{5}} + 1}{\frac{1}{2}}$$

$$= 2 \left(1 - e^{-\frac{1}{5}} \right)$$