2015 Specialist Mathematics Written examination 1 solutions

Question 1

a. $a = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

b. $\overrightarrow{OB} = \left(\sqrt{3}+1\right)\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$

$\overrightarrow{CA} = \left(\sqrt{3}-1\right)\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$

$\overrightarrow{OB}.\overrightarrow{CA} = \left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right) - 1 - 1 = 3 - 1 - 1 - 1 = 0$

The diagonals are perpendicular

Question 2

a. $R - 20g = 20 \times 1.2$

$R = 20(1.2 + 9.8) = 20 \times 11 = 220$ newtons

b. $20g - 166 = 20a$

$a = \dfrac{20 \times 9.8 - 166}{20} = \dfrac{196 - 166}{20} = 1.5$ ms^{-2}

Question 3

$\underset{\sim}{r}(t) = \left(2t^2 - 3t\right)\underset{\sim}{i} + t^2\underset{\sim}{j} - 5t\underset{\sim}{k} + \underset{\sim}{c}$

$\underset{\sim}{i} - 2\underset{\sim}{k} = \underset{\sim}{c}$

$\underset{\sim}{r}(t) = \left(2t^2 - 3t + 1\right)\underset{\sim}{i} + t^2\underset{\sim}{j} + (-5t - 2)\underset{\sim}{k}$

$\underset{\sim}{r}(2) = 3\underset{\sim}{i} + 4\underset{\sim}{j} - 12\underset{\sim}{k}$

Distance from origin $= \left|\underset{\sim}{r}(2)\right| = \sqrt{9 + 16 + 144} = 13$ metres

Question 4

a. $r^3 cis(3\theta) = 2^3 cis\left(\dfrac{\pi}{2} + 2k\pi\right)$ where k is an integer

$r = 2 \quad \theta = \dfrac{\pi}{6} + \dfrac{2k\pi}{3}$

$k = 0: \quad z_1 = 2cis\left(\dfrac{\pi}{6}\right) = 2\left(\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right) = \sqrt{3} + i$

$k = 1: \quad z_2 = 2cis\left(\dfrac{\pi}{6} + \dfrac{2\pi}{3}\right) = 2cis\left(\dfrac{5\pi}{6}\right) = 2\left(-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right) = -\sqrt{3} + i$

$k = 2: \quad z_3 = 2cis\left(\dfrac{\pi}{6} + \dfrac{4\pi}{3}\right) = 2cis\left(\dfrac{3\pi}{2}\right) = -2i$

b. $\sqrt{3} + i + 2i = \sqrt{3} + 3i$

$-\sqrt{3} + i + 2i = -\sqrt{3} + 3i$

$-2i + 2i = 0$

