

Sample solutions to the 2016 VCAA sample papers

Question 2 (Specialist Mathematics Sample examination 1)

Differentiating both sides with respect to x

$$6x + 2x \frac{dy}{dx} + 2y + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (2x + 2y) = -6x - 2y$$

$$\frac{dy}{dx} = \frac{-2(3x + y)}{2(x + y)} = \frac{-(3x + y)}{x + y}$$

When $x = 1$

$$3 + 2y + y^2 = 11$$

$$y^2 + 2y - 8 = 0$$

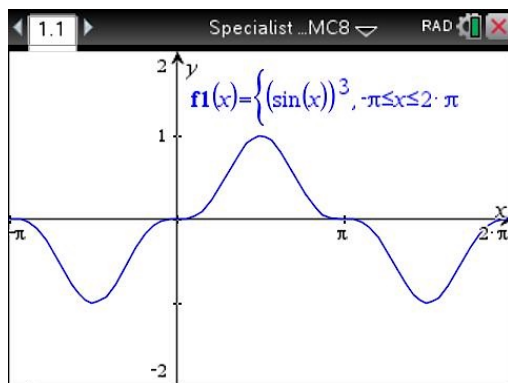
$$(y + 4)(y - 2) = 0$$

$$y = -4 \text{ or } 2$$

In the first quadrant, when $x = 1$, $y = 2$ and $\frac{dy}{dx} = -\frac{5}{3}$

Therefore, the gradient of the normal is $\frac{3}{5}$

Multiple-Choice Question 8 (Specialist Mathematics Sample examination 2)



Using symmetry, the area bounded by the curve and the x -axis is given by

$$-3 \int_{-\pi}^0 (1 - \cos^2(x)) \sin(x) dx$$

$$= -3 \int_{-1}^1 (1 - u^2)(-1) \frac{du}{dx} dx \text{ where } u = \cos(x)$$

$$= 3 \int_{-1}^1 (1 - u^2) du$$

B

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Question 6 (Mathematical Methods Sample examination 1)

a. $\log_3\left(\frac{5^2}{2}x\right) = 2$

$$\frac{25x}{2} = 9$$

$$x = \frac{18}{25}$$

b. $3x = 5 + \frac{8}{x}$ where $x = e^t$

$$3x^2 = 5x + 8$$

$$3x^2 - 5x - 8 = 0$$

$$(3x - 8)(x + 1) = 0$$

$$e^t = \frac{8}{3} \text{ as } e^t > 0 \text{ for all } t$$

$$\therefore t = \log_e\left(\frac{8}{3}\right)$$

Extended-Response Question 3a (Mathematical Methods Sample examination 2)

a. i. $X \sim \text{Bi}(20, 0.625)$

$$\Pr(X \geq 10) = 0.9153$$

ii. $\frac{\Pr(X \geq 15)}{\Pr(X \geq 10)} = \frac{0.1788}{0.9153} = 0.195$

iii. Expected value = $p = \frac{5}{8}$

$$\text{Variance} = \frac{\frac{5}{8} \times \frac{3}{8}}{20} = \frac{3}{256}$$

iv. $\Pr\left(\frac{5}{8} - \frac{2\sqrt{3}}{16} < \hat{p} < \frac{5}{8} + \frac{2\sqrt{3}}{16}\right)$
 $= \Pr(0.4085 \times 20 < X < 0.8415 \times 20)$
 $= \Pr(9 \leq X \leq 16) = 0.939$

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Module 3 Multiple-Choice Question 5 (Further Mathematics Sample examination 1)

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc} = \frac{1900^2 + 2050^2 - 2250^2}{2 \times 1900 \times 2050}$$

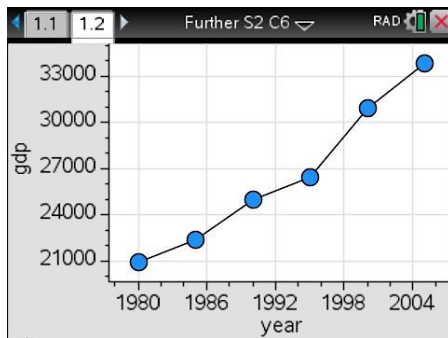
$$A = 69.3^\circ$$

$$\text{bearing} = 140^\circ + 69.3^\circ = 209.3^\circ$$

E

Core Question 6 (Further Mathematics Sample examination 2)

a.



b. Increasing positive trend

c. i. $GDP = 20,000 + 524 \times \text{time}$

c. ii. The prediction is unreliable because 2010 requires extrapolation to a value that lies outside the range of the given data.